SCS139 HW3

Friday, February 15, 2013

2:02 PM

Q1 In lecture, we found (via Ampere's law) that the magnetic field inside a solenoid is $B = \mu_0 i n$

23 mT

18 A

turns per unit length $= \frac{N}{\ell}$ ← total # turns

length ← 1.3 m

So, $N = \frac{\ell B}{\mu_0 i} \approx 1.322 \times 10^3$ turns.

$r = \frac{d}{2} = \frac{2.6 \times 10^{-2}}{2}$

The length of wire needed to form one turn is $2\pi r$.

The length of wire needed to form N turns is $2\pi r N \approx$ **108 m**.

Q2 This is a question about induction.

We should first find the magnetic flux Φ_B.

The area here is formed by a circular loop of radius r.

So, $A = \pi r^2$ and $\Phi_B = BA = B\pi r^2$.

uniform $\vec{B} \perp$ flat A

The question says the radius shrinks at a rate of 75×10^{-2} m/s.

So, $\frac{dr}{dt} = -75 \times 10^{-2}$ m/s (at the instant that the loop is released.)

This is the only variable that changes with time.

Therefore, $\frac{d\Phi_B}{dt} = \frac{d}{dt}(B\pi r^2) = \pi B\, 2r \frac{dr}{dt}$.

$r = 12 \times 10^{-2}$ m at the instant that the loop is released.

By Faraday's law, $|\mathcal{E}_{ind}| = \left|-\frac{d\Phi_B}{dt}\right| = \pi B\, 2r \left|\frac{dr}{dt}\right|$

$\approx$ **0.452 V**

Q3 For question relating to induction, we first need to find the expression for the magnetic flux:

$r = 12\text{ cm} = 12 \times 10^{-2}$ m

$\Phi_B = BA = B\pi r^2$

the radius of the loop is given; so it is circular.

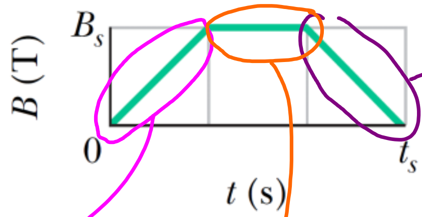
uniform $\vec{B} \perp$ flat A

In this question, only B changes with time.

Therefore,

$$\frac{d\Phi_B}{dt} = \pi r^2 \frac{dB}{dt}.$$

Next, we find $\frac{dB}{dt}$ from the plot.



During time interval [4, 6],

$$\frac{dB}{dt} = \frac{\Delta B}{\Delta t} = \frac{0 - B_s}{6-4} = -\frac{B_s}{2}$$

During time interval [0, 2),

$$\frac{dB}{dt} = \frac{\Delta B}{\Delta t} = \frac{B_s - 0}{2-0} = \frac{B_s}{2}$$

During time interval (2, 4),

$$\frac{dB}{dt} = \frac{\Delta B}{\Delta t} = 0$$

The amount of induced emf is given by Faraday's law:

$$\varepsilon_{ind} = -\frac{d\Phi_B}{dt} = -\pi r^2 \frac{dB}{dt}.$$

$$|\varepsilon_{ind}| = \pi r^2 \left|\frac{dB}{dt}\right| = \begin{cases} \pi \overset{12\text{ cm}}{r^2} \dfrac{\overset{0.5\text{ T}}{B_s}}{2} & \text{for part (a) and (c)} \\ 0 & \text{for part (b)} \end{cases}$$

$$\approx \begin{cases} 0.011\text{ V} & \text{for part (a) and (c)} \\ 0 & \text{for part (b)} \end{cases}$$

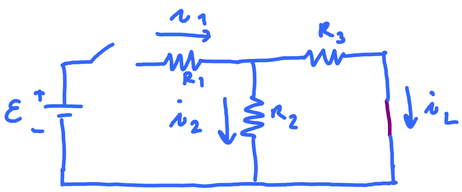
Remark: For those who care about the signs, one may answer -0.011 V for part (a) and $+0.011$ V for part (b).

Q4 Recall that there are two properties of inductor that are crucial for analyzing RL circuits with switch(es):

① When there is no change in the circuit configuration for a long time, the inductor acts like a short circuit (ordinary connecting wire).

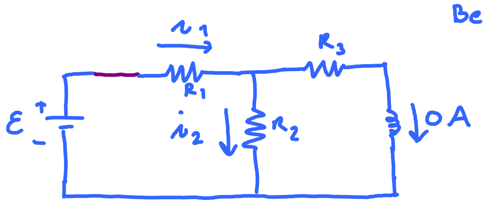
② When there is some change in the circuit (e.g. SW is open or closed), the current through the inductor cannot change abruptly.
Therefore, immediately after the change occurs, the current through the inductor must be the same as its value just before the change happens.

(a) and (b)
Here, first we consider the circuit before the SW is closed.
Assume that the SW has been in the "open" state for a long time then the inductor will act like an ordinary connecting wire:



The emf $\mathcal{E}$ is not connected.
So, we conclude that $i_1 = i_2 = i_L = 0$ A.

Now, immediately after sw is closed, by property ② above, we must have $i_L = 0$ A.

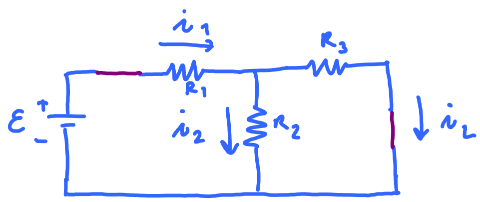


Because there is no current through the inductor, the circuit is simply a single loop with $\mathcal{E}$, R_1, R_2 all connected in series.

$$i_1 = i_2 = \frac{\mathcal{E}}{R_1 + R_2} = \frac{100}{10+20} = \frac{10}{3} \approx 3.33\,A$$

(c) and (d)

A long time later, by property ① above, the inductor becomes an ordinary wire.



$$i_1 = \frac{\mathcal{E}}{R_1 + R_2 /\!/ R_3} = \frac{\mathcal{E}}{R_1 + \frac{1}{\frac{1}{R_2}+\frac{1}{R_3}}} = \frac{50}{11} \approx 4.545\,A$$

total R

Voltage across R_2

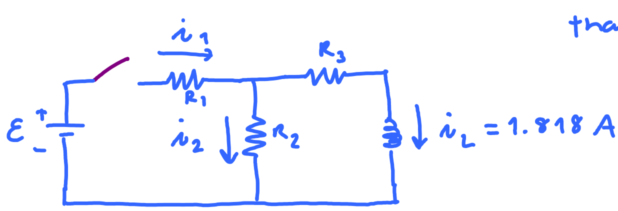
$$i_2 = \frac{i_1 (R_2 /\!/ R_3)}{R_2} = \frac{i_1 \left(1 / \frac{1}{R_2} + \frac{1}{R_3}\right)}{R_2} \approx 2.727\,A$$

$$i_L = \frac{i_1 (R_2 /\!/ R_3)}{R_3} = \frac{i_1 \left(1 / \frac{1}{R_2} + \frac{1}{R_3}\right)}{R_3} \approx 1.818\,A$$

(e) and (f)

Here, the sw is reopened.

By property ② above, immediately after the sw is reopened, i_L must be the same; that is $i_L = 1.818$ A.



The emf is disconnected from R_1. With the broken connection, there is no current through R_1:

$$i_1 = 0\,A.$$

With the sw open, note that the circuit now is simply a single loop of R_2, R_3, and L connected in series. Therefore, both R_2 and R_3 should have the same current as L. Note that the arrow for i_2 points against the direction of the current i_L. Therefore, $i_2 = -i_L \approx -1.818$ A.

(g) and (h)

A long time later, we now back to the situation discussed at the beginning for parts (a) and (b). So, $i_1 = i_2 = i_L = 0\,A$.